

Algebra

Simple Equations

When expressing problems where not all the values are known it may be necessary to construct an algebraic expression that models the problem.

A manufacture of toy dolls decided to ensure that whatever the cost of making his dolls he wanted to make £5 profit from each doll. The cost of making the doll could go up or down depending on the cost of the raw materials needed to make the doll.

The selling price would depend on the cost of making the doll plus any profit that the manufacturer wanted to make.

This could be written as:

$$\begin{array}{ccccc} \text{Selling Price} & = & \text{Cost of making doll} & + & \text{Profit} \\ \uparrow & & \uparrow & & \uparrow \\ \text{Variable} & & \text{Variable} & & \text{Fixed value} \\ & & & & (\text{£}5) \end{array}$$

This is a simple equation, the unknown variable on the left hand side of the equal sign (Selling Price) has the same value as the whole expression (Cost of making doll + Profit) on the right hand side of the equal sign.

You could just use single letters to represent the two variables above, also the profit in our example will be fixed at £5. The expression could be written in simple terms as follows:

$$S = C + 5$$

Where **S** represents the **S**elling price and **C** represents the **C**ost of making a doll.

Example 1

If it cost £10 to make a doll what is the selling price.

C now has a value of 10, so the equation becomes:

$$S = 10 + 5$$

Therefore $S = 15$

The selling price is £15.

Example 2

If it cost £8 to make a doll what is the selling price.

C now has a value of 8, so the equation becomes:

$$S = 8 + 5$$

Therefore $S = 13$

The selling price is £13.

Example 3

If it cost £12 to make a doll what is the selling price.

C now has a value of 12, so the equation becomes:

$$S = 12 + 5$$

Therefore $S = 17$

The selling price is £17

If you saw one of these dolls in a shop on sale for £20 and you knew about the equation above, it would be possible to work out what it cost to make the doll.

You know that $S = C + 5$

Note, writing the equation as $C + 5 = S$ means the same thing.

So if the Selling price is £20 you can work out the Cost of making the doll, the equation becomes:

$20 = C + 5$, therefore C must equal 15.

The cost of making the doll is £15.

Letters can therefore be used to represent number values within an equation.

Exercise 1

What is the value of C in each of the following:

- 1) $C + 12 = 20$
- 2) $C + 16 = 30$
- 3) $C + 9 = 17$
- 4) $C + 8 = 26$
- 5) $C + 15 = 42$

Exercise 2

Using the equation $S = C + 5$

- 1) If C is 24, what is the value of S .
- 2) If C is 14, what is the value of S .
- 3) If C is 7, what is the value of S .
- 4) If C is 31, what is the value of S .
- 5) If C is 29, what is the value of S .

Any letter can be chosen to represent a number, remember however that once you have assigned a number value to a letter, then anywhere that letter appears it will be replaced by the chosen number.

For example the letter 'n' is used to represent a number in the following two equations.

$$a = n + 4 \quad \text{and} \quad b = n + n + 4.$$

If 'n' takes on the value of 7 then:

$$a = 7 + 4, \quad a = 11$$

$$b = 7 + 7 + 4, \quad b = 18$$

The second equation ($b = n + n + 4$) could be written as:

$$b = 2 \times n + 4 \quad \text{or} \quad b = 2n + 4$$

↑
This means 2 x n although
you cannot see a 'x' sign

This equation will still give the correct answer when 'n' takes on a value of 7.

$$\begin{aligned} b &= 2n + 4 & \text{becomes} & & b &= 2 \times 7 + 4 \\ b &= 14 + 4 \\ b &= 18 \end{aligned}$$

Exercise 3

If 'n' takes on the value of 6 find what value 'b' represents.

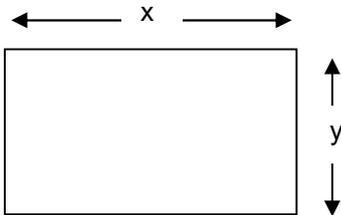
- 1) $b = 2n + 6$
- 2) $b = 3n + 3$
- 3) $b = 4n + 5$
- 4) $b = 7n + 9$
- 5) $b = 10n + 7$

The value of 'b' above depends on the value of one variable 'n' .

Working with two variables

Sometimes there are two variables involved in an equation.

Consider the perimeter of the rectangle below.



The perimeter (distance all the way around the rectangle) will be found by adding the lengths of all the sides.

Let the letter 'p' represent the perimeter.

$$p = x + x + y + y$$

These are called terms and like terms (those with the same letter) have been grouped together.

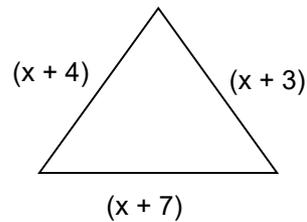
$$p = 2x + 2y$$

Exercise 4

$$p = 2x + 2y$$

- 1) If 'x' takes on the value of 4 and 'y' takes on the value 3, find the perimeter 'p'.
- 2) If 'x' takes on the value of 5 and 'y' takes on the value 8, find the perimeter 'p'.
- 3) If 'x' takes on the value of 9 and 'y' takes on the value 10, find the perimeter 'p'.
- 4) If 'x' takes on the value of 12 and 'y' takes on the value 6, find the perimeter 'p'.
- 5) If 'x' takes on the value of 13 and 'y' takes on the value 7, find the perimeter 'p'

Perimeter of triangles



This drawing is
Not To Scale
(N.T.S.)

$$\begin{aligned}\text{Perimeter} &= (x + 4) + (x + 3) + (x + 7) \\ &= 3x + 14\end{aligned}$$

If you are given a value for the perimeter it is possible to work out the value of 'x'.

E.g. if the letter 'p' is used to represent the perimeter and its value is 47, the above equation would be written as:

$$\begin{aligned}p &= 3x + 14 \\ 47 &= 3x + 14\end{aligned}$$

to find 'x' we need to get 'x' on one side of the equation all on its own.

To remove the 14 subtract 14 from both sides of the equation.

$$\begin{aligned}47 - 14 &= 3x + 14 - 14 \\ 33 &= 3x \\ 11 &= x \\ \text{or } x &= 11\end{aligned}$$

This is a golden rule, whatever mathematical operation is carried out on one side of the equation must be carried out on the other.

Now that a value for 'x' has been found you can work out the lengths of the triangle.

$$\begin{aligned}(x + 4) &= 11 + 4 = 15 \\ (x + 3) &= 11 + 3 = 14 \\ (x + 7) &= 11 + 7 = 18\end{aligned}$$

Adding these 3 lengths together gives:

$$15 + 14 + 18 = 47$$

Exercise 5

Find the value of 'x' for the following triangles and the length of the sides.

