

Factors and Multiples

A **factor** of a number is a number that goes into that number exactly:

The factors of 16 are: 1, 2, 4, 8, 16
 $16 = 1 \times 16 = 2 \times 8 = 4 \times 4$

To factorise a number means to write it as a product.
Note the word 'product' implies write as the multiplication of two numbers.

In writing 16 as 2×8 we have factorised the number 16.

A **multiple** of a number is a number which that number goes into exactly:

The multiples of 8 are 8, 16, 24, 32, ...
Multiplying the number under consideration by a whole number makes multiples of that number.

The **Highest Common Factor** of two or more numbers is the highest number that is a factor of both numbers.

12 is the **HCF** of 24 and 36

The **Lowest Common Multiple** of two or more numbers is the smallest number that is a multiple of both numbers.

15 is the **LCM** of 3 and 5

Rules for factors

2 is a factor of any even number (last digit is divisible by two)

3 is a factor of a number if its digit sum is divisible by 3
e.g. 573: $5 + 7 + 3 = 15$, $1 + 5 = 6$ 1242: $1 + 2 + 4 + 2 = 9$

5 is a factor of any number ending in 5 or 0

9 is factor of a number if its digit sum is 9
e.g. 4734: $4 + 7 + 3 + 4 = 18$, $1 + 8 = 9$

10 is a factor of any number ending in 0

11 is a factor if a 3 digit figure if its middle digit is the sum of the two outside digits
e.g. 693: two outside digits when added together equal the middle digit

Prime Factorisation

To factorise a number, write it as a product of its prime numbers only. Start by dividing the number under consideration by the first prime number '2', continue dividing by 2 as far as possible. Repeat the process with the next prime number '3' and so on until you have broken the number into a product of primes.

Consider 28

$$2 \times 14 = 28$$

$$2 \times 2 \times 7 = 28 \text{ (this represents 28 as a product of its primes)}$$

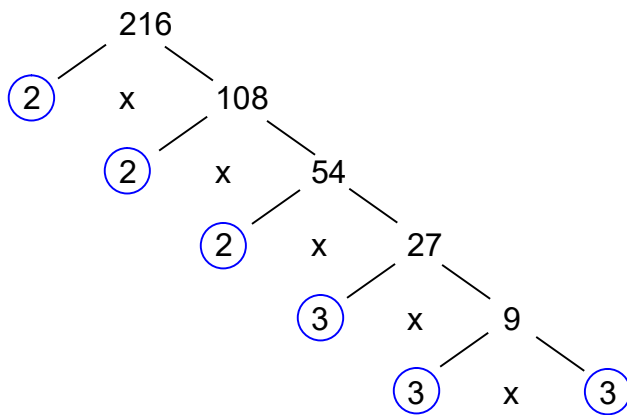
Consider 36

$$2 \times 18 = 36$$

$$2 \times 2 \times 9 = 36$$

$$2 \times 2 \times 3 \times 3 = 36 \text{ (this represents 36 as a product of its primes)}$$

Consider 216



$$2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216 \text{ (this represents 216 as a product of its primes)}$$

Finding the HCF and LCM using prime factorisation

When looking for the HCF or LCM of two numbers it is best to write each number as a product of its primes and look for common factors.

Consider 60 and 210:

$$60 = 2 \times 2 \times 3 \times 5$$

$$210 = 2 \times 3 \times 5 \times 7$$

by inspection you will see that $2 \times 3 \times 5$ are common to both numbers, therefore the **Highest Common Factor (HCF)** will be 30

A multiple of both numbers must include all the prime factors of both numbers, but you need only use common factors once.

$$60 = 2 \times 2 \times 3 \times 5$$

$$210 = 2 \times 3 \times 5 \times 7$$

The **Lowest Common Multiple (LCM)** of 60 and 210 will be

$$2 \times 2 \times 3 \times 5 \times 7 = 420$$

Find the H.C.F of

- a) 12, 15,21
- b) 16, 24, 40
- c) 28, 70,120,160
- d) 35, 38,42
- e) 96,120,144

Find the L.C.M of

- f) 5,6,8
- g) 20,30
- h) 7,9,12
- i) 100,150,235
- j) 96,120,144

Answers

- | | |
|-------------------|----|
| a) 12, 15,21 | 3 |
| b) 16, 24, 40 | 8 |
| c) 28, 70,120,160 | 2 |
| d) 35, 28,42 | 1 |
| e) 96,120,144 | 24 |

- | | |
|----------------|-------|
| f) 5,6,8 | 120 |
| g) 20,30 | 60 |
| h) 7,9,12 | 252 |
| i) 100,150,235 | 14100 |
| j) 96,120,144 | 1440 |