

FRACTIONS



TOPIC	CURRICULM REFERENCE	PAGE
DEFINITION	N2/E2.1	2
EQUIVALENT FRACTIONS / REDUCING TO LOWEST TERM	N2/E3.2	3
WORDS USED / NUMERATOR & DENOMINATOR	N2/L1.1	4
TYPES OF FRACTION		5
CONVERTING IMPROPER (TOP HEAVY) FRACTIONS INTO MIXED NUMBERS		6
LOWEST COMMON MULTIPLE (LCM)	N2/L2	7
ORDERING FRACTIONS	N2/L2	8
ADDITION	N2/L2	9
SUBTRACTION	N2/L2	10
ADDITION (Without necessarily using the LCM)	N2/L2	11
COMBINED ADDTION AND SUBTRACTION	N2/L2	12
MULTIPLICATION & CANCELLING		13
DIVISION		14
ADDITION OF MIXED NUMBERS		15
SUBTRACTION OF MIXED NUMBERS		16
SUBTRACTION OF MIXED NUMBERS (continued)		17
MULTIPLICATION OF MIXED NUMBERS		18
DIVISION OF MIXED NUMBERS		19
CALCULATING FRACTIONS OF WHOLE NUMBERS	N2/L1	20

FRACTIONS - DEFINITION

Fractions are ways of expressing a proportion or part of something.

Examples

One whole square can be divided into 2 equal parts.

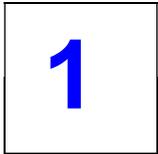


Fig 1

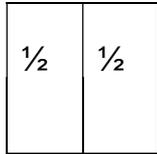


Fig 2

Figure 1 shows 1 whole unit, in this case the unit is a square.

Figure 2 shows 1 whole unit divided into 2 equal parts, each of the parts represents $\frac{1}{2}$ of the whole unit.

One whole square can be divided into 4 equal parts.

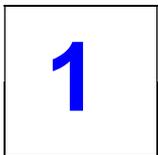


Fig 3

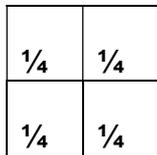


Fig 4

Figure 3 shows 1 whole unit, in this case the unit is a square.

Figure 4 shows 1 whole unit divided into 4 equal parts, each of the parts represents $\frac{1}{4}$ of the whole unit.

Exercise 1

One whole square can be divided into 16 equal parts.

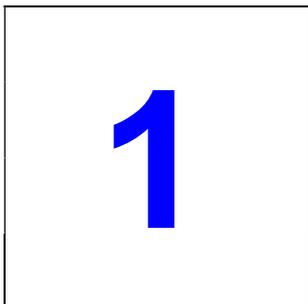


Fig 5

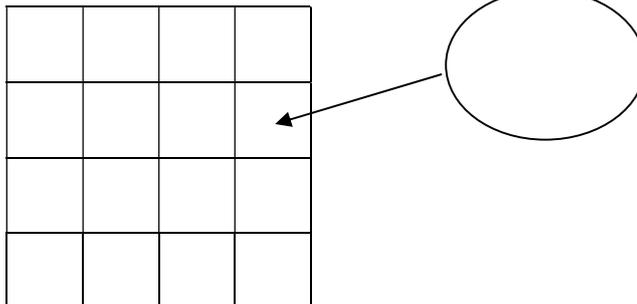


Fig 6

Figure 5 shows 1 whole unit, in this case the unit is a large square.

Figure 6 shows 1 whole unit divided into equal parts, each of the parts represents of the whole unit.

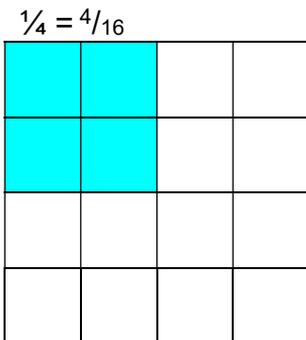
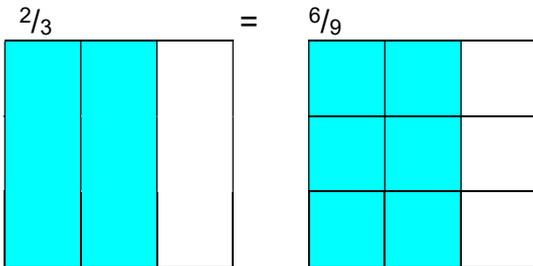
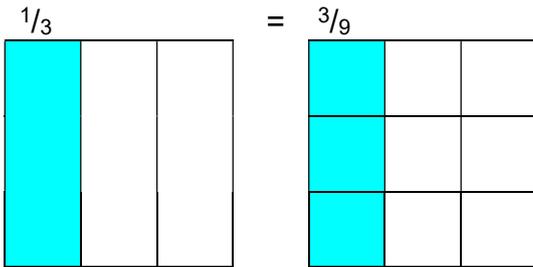
Other examples of where fractions are used in everyday life.

Fuel tank indicator – half full, Clock – quarter past, Capacity - half a litre, Rulers – half a cm, Days of the week, Months of the year, Money.

FRACTIONS – EQUIVALENT FRACTIONS / REDUCING TO LOWEST TERM

Examples

1							
$\frac{1}{2}$				$\frac{1}{2}$			
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{8}$							



Multiplying top and bottom number by the same non zero number, does not alter the fraction.

Reducing fraction to its lowest form

Divide top and bottom by the same number as far as you can go.

Exercise 2

Reduce the following to their lowest form.

1) $\frac{4}{6} =$

2) $\frac{3}{9} =$

3) $\frac{5}{10} =$

4) $\frac{6}{12} =$

FRACTIONS – WORDS USED / NUMERATOR & DENOMINATOR

Fractions in words

To find a half of something divide by 2.
To find a third of something divide by 3.
To find an eighth of something divide by 8.
To find a tenth of something divide by 10.

Examples

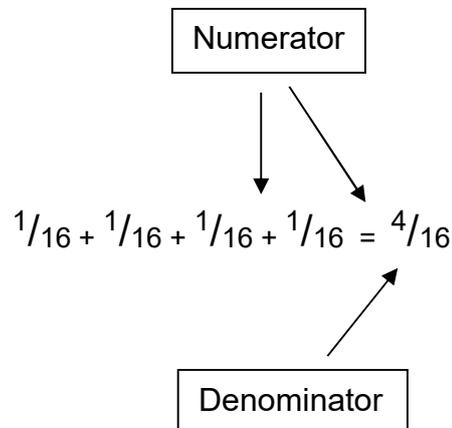
One half of £10 = $£10 \div 2$ = £5
One third of 12 = $12 \div 3$ = £4

Exercise 3

- 1) Find one tenth of 90.
- 2) Find one sixth of 24.
- 3) Find one third of 27.

Numerator / Denominator

$\frac{1}{16}$			



The **top** number of a fraction is called the **numerator**.
The **bottom** number of a fraction is called the **denominator**.

The **numerator** value represents the **number** of fraction parts you are considering.

The **denominator** denotes how many equal parts the whole has been broken into.

The fraction $\frac{4}{16}$ represents 4 parts, the size of each part is one sixteenth of the whole.

It may help to think of money, someone can give you £1 in different denominations.

£1 = 10 x 10p in this case the denominations are 10p pieces

£1 = 5 x 20p in this case the denominations are 20p pieces

£1 = 2 x 50p in this case the denominations are 50p pieces

(See coins sheet)

FRACTIONS – TYPES OF FRACTION

Vulgar fractions (Common / Proper)

The top number is always smaller than the bottom number.

Examples

$\frac{1}{2}$, $\frac{3}{8}$, $\frac{7}{9}$, $\frac{9}{10}$,

Improper (Top heavy)

The top number is always larger than the bottom number.

Examples

$\frac{4}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{11}{4}$,

What happens if both top and bottom number are the same?

Examples

$\frac{3}{3}$, $\frac{5}{5}$, $\frac{7}{7}$, $\frac{4}{4}$,

Consider $\frac{5}{5}$, the number at the bottom tells us how many parts make a whole.

1				
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Therefore $\frac{5}{5} = 1$

If both top and bottom are the same number the fraction will equal 1 and is no longer a fraction.

Back to Improper (Top heavy)

Using same examples as above

$\frac{4}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{11}{4}$,

Improper fractions can be simplified by dividing the bottom number (denominator) into the top number (numerator), leaving any remainder as a fraction.

Examples

Consider $\frac{6}{5}$

$$\begin{aligned} 6 \div 5 &= 1 \text{ remainder } 1 \text{ fifth} \\ &= 1\frac{1}{5} \end{aligned}$$

1					
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$1\frac{1}{5}$ is called a mixed number, it contains a whole number and a fraction.

FRACTIONS – CONVERTING IMPROPER (TOP HEAVY) FRACTIONS INTO MIXED NUMBERS

Example

Divide the bottom number (denominator) into the top number (numerator), leave any remainder as a fraction.

Consider $11/4$

$$11 \div 4 = 2 \text{ remainder } 3 \text{ quarters}$$

$$= 2 \frac{3}{4}$$

Exercise 4

Change the following improper fractions into mixed numbers.

1) $4/3$

2) $8/7$

3) $9/3$

4) $10/4$

Converting Mixed numbers into Improper (top heavy) fractions

Sometimes you may need to convert a mixed number into an improper fraction.

Consider $2 \frac{3}{4}$, the 4 at the bottom means that there are 4 quarters to each whole.

$$2 \frac{3}{4} = 2 + \frac{3}{4}$$

$$2 \frac{3}{4} = 1+1 + \frac{3}{4}$$

Now if you replace the two number ones with their equivalent quarters you have:

$$2 \frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$

$$2 \frac{3}{4} = \frac{11}{4}$$

Quick way – multiply whole number by the denominator, add this result to the numerator and put this final figure over the denominator.

Consider $2 \frac{3}{4}$ again, this time using above quick method becomes, $(2 \times 4) + 3$ all over 4
i.e. $11/4$.

Exercise 5

Change the following mixed numbers into improper fractions.

1) $2 \frac{2}{3}$

2) $3 \frac{3}{7}$

3) $5 \frac{1}{4}$

4) $4 \frac{1}{6}$

FRACTIONS – LOWEST COMMON MULTIPLE (LCM)

The LCM of any two numbers is the smallest number that both of them will go into.

Examples

Consider the numbers 7 and 9.

The multiples of 7 are: 7, 14, 21, 28, 35, 42, 49, 56, **63**, 70

The multiples of 9 are: 9, 18, 27, 36, 45, 54, **63**, 72, 81, 90

The number **common** to both lines of multiples is **63**, that is **63** can be divided exactly with no remainder by either 7 or 9.

Consider the numbers 6 and 8.

The multiples of 6 are: 6, 12, 18, **24**, 30, 36, 42, **48**, 54, 60

The multiples of 8 are: 8, 16, **24**, 32, 40, **48**, 56, 64, 72, 80

There are two sets of numbers **common** to both lines of multiples; the numbers are **24** and **48**.

24 is **lower** than **48** so this is the number chosen.

So the Lowest Common Multiple (LCM) of 6 and 8 is **24**.

Exercise 6

Find the Lowest Common Multiple of:

- 1) 3 & 7
- 2) 4 & 6
- 3) 6 & 9
- 4) 2, 3 & 5
- 5) 3, 5 & 10

FRACTIONS – ORDERING FRACTIONS

Comparing fractions of with the same denominator is easy.

Consider $\frac{1}{6}$ and $\frac{3}{6}$, as you are comparing similar fractions it is obvious that $\frac{3}{6}$ is larger than $\frac{1}{6}$.

However, when comparing two fractions with different denominators, it can be difficult to know which one is largest or smallest.

Example

Consider $\frac{2}{3}$ and $\frac{3}{5}$ - the denominators are different, therefore you cannot compare them.

The denominators of the two fractions need to be the same. To do this you need to know the LCM of the two denominators.

The multiples of 3 are: 3, 6, 9, 12, **15**, 18 ...

The multiples of 5 are: 5, 10, **15**, 20, 25 ...

The LCM of 3 and 5 is **15**.

Looking back at the fractions ($\frac{2}{3}$ and $\frac{3}{5}$), these must now be re-written with a denominator of 15.

In the case of the first fraction the denominator will need to be multiplied by 5, in the case of the second fraction the denominator will need to be multiplied by 3.

To keep each fraction at its original value, you must also multiply the numerator by the same value as you have multiplied the denominator.

So,

$$\frac{2}{3} = \frac{10}{15} \quad (\text{top and bottom multiplied by 5})$$

&

$$\frac{3}{5} = \frac{9}{15} \quad (\text{top and bottom multiplied by 3})$$

Now you can see that $\frac{10}{15}$ is bigger than $\frac{9}{15}$.

Therefore $\frac{2}{3}$ is bigger than $\frac{3}{5}$.

Exercise 7

Which of each pair of fractions is the largest?

1) $\frac{5}{6}$, $\frac{3}{4}$

2) $\frac{3}{5}$, $\frac{7}{10}$

3) $\frac{7}{12}$, $\frac{3}{4}$

4) $\frac{3}{8}$, $\frac{2}{5}$

5) $\frac{7}{8}$, $\frac{6}{10}$

FRACTIONS – ADDITION

There are everyday situations where you may need to add fractions.

Example

If there are two pints of milk in the fridge, one is half ($\frac{1}{2}$) full and the other is two thirds ($\frac{2}{3}$) full, how much milk is there altogether.

To solve this problem you need to add two fractions, in this case $\frac{1}{2} + \frac{2}{3}$.

The denominators are not the same, so the first thing you must do is to change the fractions into the same type. To do this you work out the LCM for both denominators.

The LCM for 2 and 3 is 6. (if unsure about this review page 6)

For the first fraction $\frac{1}{2}$ you have had to multiply the denominator by 3 to get to 6, therefore you must multiply the numerator by 3 also so that the fraction value remains unchanged.

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

In the same way for the second fraction you multiply the denominator by 2 to get to 6, therefore you must multiply the numerator by 2. The fraction value then remains unchanged.

$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Now you can add both fractions.

$$\frac{3}{6} + \frac{4}{6}$$

When both of the denominators are the same you can re-write the sum as:

$$\frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Exercise 8

Add the following.

1) $\frac{5}{6} + \frac{3}{4}$

2) $\frac{3}{5} + \frac{7}{10}$

3) $\frac{7}{12} + \frac{3}{4}$

4) $\frac{3}{8} + \frac{2}{5}$

5) $\frac{7}{8} + \frac{6}{10}$

FRACTIONS – SUBTRACTION

Sometimes you may need to know how to subtract one fraction from another.

Example

A runner had decided to run three quarters ($\frac{3}{4}$) of a mile, after running for two thirds ($\frac{2}{3}$) of a mile he became tired and wondered how much more he would have to run.

To solve this problem you need to subtract the smaller fraction from the larger fraction. At this stage you might not know which fraction is the smallest.

The denominators are not the same, so the first thing you must do is to change the fractions into the same type. To do this you work out the LCM for both denominators.

The LCM for 4 and 3 is 12. (if unsure about this review page 7).

For the first fraction ($\frac{3}{4}$) you have had to multiply the denominator by 3 to get to 12, therefore you must multiply the numerator by 3. The fraction value then remains unchanged.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

In the same way for the second fraction you multiply the denominator by 4 to get to 12, therefore you must multiply the numerator by 2. The fraction value then remains unchanged.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Now you can see which is the smaller fraction.

The smaller fraction can be subtracted from the larger fraction.

$$\frac{9}{12} - \frac{8}{12}$$

When both of the denominators are the same you can re-write the sum as:

$$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

The runner had $\frac{1}{12}$ th of a mile to go.

Exercise 9

Work out the following.

1) $\frac{5}{6} - \frac{1}{4}$

2) $\frac{3}{5} - \frac{2}{7}$

3) $\frac{7}{9} - \frac{2}{3}$

4) $\frac{3}{8} - \frac{1}{5}$

5) $\frac{2}{3} - \frac{1}{5}$

FRACTIONS – ADDITION (WITHOUT NECESSARILY USING THE LCM)

Working out the LCM can be tedious; here is a quicker approach to solving addition and subtraction of problems without necessarily using the LCM.

Example

Consider $\frac{2}{3} + \frac{3}{5}$

Multiplying both denominators by each other (3 x 5 to give 15), will immediately give a common denominator (15) that both 3 and 5 will divide into.

Now you continue as normal.

For the first fraction $\frac{2}{3}$ you have had to multiply the denominator by 5 to get to 15, therefore you must multiply the numerator by 5 also so that the fraction value remains unchanged.

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

In the same way for the second fraction you multiply the denominator by 3 to get to 15, therefore you must multiply the numerator by 3 also so that the fraction value remains unchanged.

$$\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

Now you can add both fractions.

$$\frac{10}{15} + \frac{9}{15}$$

When both of the denominators are the same you can re-write the sum as:

$$\frac{10 + 9}{15} = \frac{19}{15} = 1\frac{4}{15}$$

Exercise 10

Solve the following using the above method for find a multiple that both denominators will divide into.

1) $\frac{1}{6} + \frac{3}{4}$

2) $\frac{3}{5} + \frac{8}{9}$

3) $\frac{7}{10} + \frac{3}{4}$

4) $\frac{3}{7} + \frac{2}{5}$

5) $\frac{7}{8} + \frac{2}{3}$

FRACTIONS – COMBINED ADDITION AND SUBTRACTION

There are some situations that arise where both addition and subtraction of fractions may be necessary.

Example

Mary wanted to make a cake but did not have any flour. She visited her two neighbours who both gave her all they had, one neighbour gave $\frac{3}{4}$ lb of flour and the other $\frac{2}{3}$ lb of flour.

Mary returned home and started to make the cake when one neighbour knocked on her door, the neighbour asked if she could just have $\frac{1}{6}$ lb of flour back for her daughter's school lesson the following day.

Mary needed to work out if she had enough flour for the recipe, which needed 1lb of flour.

To solve this you need to both add and subtract fractions.

$$\frac{3}{4} + \frac{2}{3} - \frac{1}{6}$$

You now have three fractions. Even so you can work as normal, first find the common denominator of all the fractions and re-write the sum.

A common denominator is 72 ($4 \times 3 \times 6$), the Lowest Common Denominator is 12

The sum can now be written as:

$$\frac{9 + 8 - 2}{12} = \frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$$

Exercise 11

Work out the following.

1) $\frac{7}{9} - \frac{2}{3} + \frac{1}{6}$

2) $\frac{3}{5} + \frac{2}{7} - \frac{1}{7}$

3) $\frac{2}{3} - \frac{1}{5} + \frac{2}{10}$

4) $\frac{3}{8} - \frac{1}{5} + \frac{1}{10}$

5) $\frac{5}{6} + \frac{1}{4} - \frac{1}{5}$

FRACTIONS – MULTIPLICATION

When it is necessary to represent one fraction as a proportion of another, you will need to multiply one fraction by the other.

Example

John had already drunk $\frac{1}{4}$ of a pint of milk when his cat looked up him with begging eyes. John decided to tip $\frac{1}{2}$ of his milk into a saucer for his cat, how much did he have left.

To solve this problem you need to express it as a multiplication of two fractions.

In this case John would be left with $\frac{1}{2}$ of $\frac{3}{4}$ of a pint of milk, this would be written as $\frac{1}{2} \times \frac{3}{4}$.

When multiplying fractions you simply multiply the numerators and then the denominators.

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

John is left with $\frac{3}{8}$ of a pint of milk.

Cancelling down

Sometimes you may be able to simplify the fraction problem by cancelling.

Consider $\frac{2}{5} \times \frac{5}{6}$

If the same number appears on the top and bottom they cancel each other out, in the problem above there is a 5 on both the top and bottom. The problem can therefore be simplified as follows.

$$\frac{2}{\cancel{5}_1} \times \frac{\cancel{5}^1}{6} = \frac{2}{6} \leftarrow \text{this fraction can be reduced to its lowest term (see page 3 if unsure about this)} \rightarrow \frac{1}{3}$$

You might be able to do all of these operations in one go.

Consider $\frac{3}{10} \times \frac{10}{12}$, below is one way of cancelling down and simplifying.

$$\frac{\cancel{3}^1}{\cancel{10}_1} \times \frac{\cancel{10}^1}{\cancel{12}_4} = \frac{1}{4}$$

Exercise 12

Work out the following.

1) $\frac{4}{5} \times \frac{3}{4}$

2) $\frac{3}{5} \times \frac{8}{9}$

3) $\frac{7}{10} \times \frac{3}{4}$

4) $\frac{3}{7} \times \frac{2}{5}$

5) $\frac{7}{8} \times \frac{2}{3}$

FRACTIONS – DIVISION

Occasionally you may need to divide one fraction by another.

Example

Julie is wants to wedge the garden gate open while she puts some rubbish in the car.

The gap between the bottom of the gate and the floor is $\frac{3}{4}$ cm, Julie finds some small pieces of wood in the garage which are about $\frac{1}{8}$ cm thick.

Julie wonders how many pieces of wood she will need to fill the gap.

What Julie needs to know, is how many eighths' go into three-quarters.

The sum becomes:

$$\frac{3}{4} \div \frac{1}{8}$$

To calculate this division the second fraction is turned upside down, then the two fractions are multiplied to provide the answer.

$$\frac{3}{4} \div \frac{1}{8}$$

Note, the division sign has changed into a multiplication sign.

$$\text{is written as } \frac{3}{4} \times \frac{8}{1} = \frac{24}{4} = \frac{6}{1} = 6$$

Julie would need 6 pieces of wood to wedge the gate open, that's all there is to it.

Don't forget that once you start multiplying out fractions you may be able to simplify them by cancelling down as shown on page 13 above.

Exercise 13

1) $\frac{3}{4} \div \frac{1}{4}$

2) $\frac{8}{9} \div \frac{3}{5}$

3) $\frac{7}{10} \div \frac{3}{4}$

4) $\frac{6}{7} \div \frac{2}{5}$

5) $\frac{7}{8} \div \frac{2}{3}$

FRACTIONS – ADDITION OF MIXED NUMBERS

There are occasions when you will need to add numbers that are a combination of fractions, whole numbers, and mixed numbers.

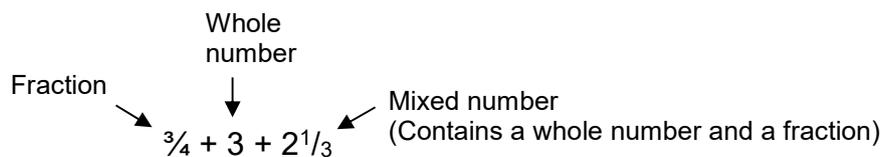
Example

David had to deliver bread to three villages, the first village was $\frac{3}{4}$ mile from his home.

The distance between the first and second villages was 3 miles, the distance between the second and third villages was $2\frac{1}{3}$ miles.

What was the total distance from his home to the furthest village.

To solve this problem you need to add all of the distances.



You can first add all the whole numbers.

$$1) \quad 3 + 2 = 5$$

Then add the fractions:

$$2) \quad \frac{3}{4} + \frac{1}{3}$$

$$\frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}$$

Then add together the above results.

$$3) \quad 5 + 1\frac{1}{12} = 6\frac{1}{12} \text{ miles}$$

Exercise 14

Add the following.

$$1) \quad 2\frac{1}{4} + 1\frac{3}{4}$$

$$2) \quad 3\frac{3}{5} + 3\frac{2}{5} + 2$$

$$3) \quad 5\frac{5}{12} + 2\frac{1}{6} + \frac{5}{12}$$

$$4) \quad \frac{3}{8} + \frac{2}{5} + 6$$

$$5) \quad 4\frac{7}{8} + 1\frac{6}{10} + 3$$

FRACTIONS – SUBTRACTION OF MIXED NUMBERS – Method 1

There are occasions when you will need to subtract numbers that are a combination of fractions, whole numbers, and mixed numbers.

Example

June weighed herself at the start of the week and she was $9\frac{1}{2}$ st, by the end of the week she had lost $1\frac{3}{4}$ st. What was her new weight?

To solve this problem, first you must convert all numbers to top heavy (improper) fractions. Then you take the smaller fraction from the larger one.

$$9\frac{1}{2} = \frac{19}{2}$$

$$1\frac{3}{4} = \frac{7}{4}$$

The denominators are different so you need to make them both the same. Multiplying the numerator and denominator of $\frac{19}{2}$ by 2 will give $\frac{38}{4}$.

Now you can subtract the smaller fraction from the larger.

$$\frac{38}{4} - \frac{7}{4} \quad \text{can be written as,}$$

$$\frac{38}{4} - \frac{7}{4} = \frac{31}{4} = 7\frac{3}{4} \text{ st}$$

Exercise 15

Subtract the following.

1) $2\frac{1}{4} - 1\frac{3}{4}$

2) $3\frac{2}{3} - 3\frac{1}{5}$

3) $5\frac{1}{10} - 2\frac{1}{5}$

4) $4\frac{3}{8} - 1\frac{1}{4}$

5) $4\frac{5}{6} - 2\frac{2}{3}$

FRACTIONS – SUBTRACTION OF MIXED NUMBERS – Method 2

It is possible to work with the whole number part of a mixed number when performing subtraction. You may however need to borrow from the whole number when you end up having to take a large fraction from a smaller one.

Example (no borrowing needed)

$$3\frac{3}{4} - 1\frac{1}{2}$$

Subtract the whole numbers first.

$$3 - 1 = 2 \quad \leftarrow$$

First part: result of working with whole numbers

Then subtract the fractional parts of the whole numbers.

$$\frac{3}{4} - \frac{1}{2}$$

becomes

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4} \quad \leftarrow$$

Second part: result of working with fractions

Now add the two results.

$$2 + \frac{1}{4} = 2\frac{1}{4} \quad \leftarrow$$

Third part: add both of the above result together

$$3\frac{3}{4} - 1\frac{1}{2} = 2\frac{1}{4}$$

Example (borrowing needed)

$$3\frac{3}{4} - 1\frac{7}{8}$$

Subtract the whole numbers first.

$$3 - 1 = 2$$

Then subtract the fractional parts of the whole numbers.

$$\frac{3}{4} - \frac{7}{8} = \frac{6}{8} - \frac{7}{8}$$

You cannot take $\frac{7}{8}$ from $\frac{6}{8}$ without introducing negative fractions because $\frac{7}{8}$ is larger than $\frac{6}{8}$ so you must borrow one whole unit from the previous result, here are the steps:

$$2 \quad + \quad (\frac{6}{8} - \frac{7}{8})$$

$$1 + 1 \quad + \quad (\frac{6}{8} - \frac{7}{8}) \quad \text{but 1 whole unit is the same as } \frac{8}{8}, \text{ so you can replace the 1 with } \frac{8}{8}.$$

$$1 + \quad (\frac{8}{8} + \frac{6}{8} - \frac{7}{8})$$

$$1 + \quad (\frac{14}{8} - \frac{7}{8})$$

$$1 + \frac{7}{8} = 1\frac{7}{8}$$

Exercise – Repeat the previous exercise (15) but using the method above.

(Note: for some questions negative fractions cannot be avoided if there is no whole number to borrow from i.e. $\frac{3}{5} - \frac{4}{5} = -\frac{1}{5}$)

FRACTIONS – MULTIPLICATION OF MIXED NUMBERS

There are occasions when you will need to multiply numbers that are a combination of fractions, whole numbers, and mixed numbers.

Example

Sam has planted a sunflower, it is already $3\frac{1}{2}$ cm tall. Sam's friend said it will more than double its size every week, in fact to be accurate it will grow at a rate of $2\frac{1}{3}$ every week. How big will it be after 1 week's growth?

This problem is expressed as the multiplication of the two mixed numbers.

$$3\frac{1}{2} \times 2\frac{1}{3}$$

To solve this problem, first you must convert all numbers to top heavy (improper) fractions. Then multiply both numbers together.

$$3\frac{1}{2} = \frac{7}{2}$$

$$2\frac{1}{3} = \frac{7}{3}$$

$$\frac{7}{2} \times \frac{7}{3} = \frac{49}{6}$$

$$= 8\frac{1}{6}$$

Exercise 16

Multiply the following.

1) $2\frac{1}{4} \times 1\frac{3}{4}$

2) $3\frac{2}{3} \times 3\frac{1}{5}$

3) $5\frac{1}{10} \times 2\frac{1}{5}$

4) $4\frac{3}{8} \times 1\frac{1}{4}$

5) $4\frac{5}{6} \times 2\frac{2}{3}$

FRACTIONS – DIVISION OF MIXED NUMBERS

There are occasions when you will need to divide numbers that are a combination of fractions, whole numbers, and mixed numbers.

Example

Jean has made $4\frac{1}{2}$ litres of punch for her party, she would be wondering how many $\frac{3}{4}$ litre glasses she could fill.

This problem is expressed as the division of the two numbers.

$$4\frac{1}{2} \div \frac{3}{4}$$

To solve this problem, first you must convert all mixed numbers to top heavy (improper) fractions.

Then turn upside down the smaller fraction and multiply it by the larger fraction. This is the same method for dividing any one fraction by another.

$$4\frac{1}{2} = \frac{9}{2}$$

The fraction $\frac{3}{4}$ is already in the correct form

The problem now becomes:

$$\frac{9}{2} \div \frac{3}{4}$$

To divide $\frac{3}{4}$ into $\frac{9}{2}$ you must turn $\frac{3}{4}$ upside down and multiply the two fractions together.

$$\begin{aligned}\frac{9}{2} \times \frac{4}{3} &= \frac{36}{6} \\ &= 6\end{aligned}$$

Jean can fill 6 glasses from her supply of punch.

Exercise 17

Solve the following.

1) $2\frac{1}{4} \div 1\frac{1}{8}$

2) $6\frac{2}{3} \div 3\frac{2}{6}$

3) $5\frac{1}{10} \div 2\frac{1}{5}$

4) $4\frac{3}{8} \div 1\frac{1}{4}$

5) $4\frac{5}{6} \div 2\frac{2}{3}$

FRACTIONS - CALCULATING FRACTIONS OF WHOLE NUMBERS

There are occasions when you will need to calculate fractions of whole numbers.

Example

A car rally was planned to be 600 miles long.

However due to bad weather, the organisers decided it should only be two thirds of the original distance.

To solve this problem, you multiply the two numbers together. The whole number is written as a top heavy fraction with a denominator of one.

$$\frac{600}{1} \times \frac{2}{3} = \frac{1200}{3}$$

$$= 400$$

The new distance for the car rally is 400 miles.

Exercise 18

Multiply the following.

1) $20 \times \frac{3}{4}$

2) $30 \times \frac{1}{5}$

3) $56 \times \frac{1}{5}$

4) $\frac{1}{4} \times 48$

5) $\frac{5}{6} \times 24$

FRACTIONS – WORDED QUESTIONS

Use examples from textbooks.

Finding the LCM for larger numbers.

It may not be obvious what the LCM is for larger numbers, consider 128 and 208.

Writing down the multiples of both numbers until the LCM is found would be rather long winded.

It is best to write each number as the product of its prime factors, then pick out a product that includes all the factors from both numbers.

128:

$$2 \times 64$$

$$2 \times 2 \times 32$$

$$2 \times 2 \times 2 \times 16$$

$$2 \times 2 \times 2 \times 2 \times 8$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 4$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \text{ (product of primes)}$$

$$= 2^7$$

208:

$$2 \times 104$$

$$2 \times 2 \times 52$$

$$2 \times 2 \times 2 \times 26$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 13 \text{ (product of primes)}$$

$$= 2^4 \times 13$$

LCM in this case will be $= 2^7 \times 13$

Consider 108 and 324

108:

$$2 \times 54$$

$$2 \times 2 \times 27$$

$$2 \times 2 \times 3 \times 9$$

$$2 \times 2 \times 3 \times 3 \times 3$$

$$2^2 \times 3^3$$

324:

$$2 \times 162$$

$$2 \times 2 \times 81$$

$$2 \times 2 \times 3 \times 27$$

$$2 \times 2 \times 3 \times 3 \times 9$$

$$2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$2^2 \times 3^4$$

$$\text{LCM} = 2^2 \times 3^4$$

Don't need 3^3 as it is included in 3^4 .